(1) A Family of Functions has the form $y = ax^b \ln x$ where a and b are non-zero constants. Calculate the value of a and the value of b so that this function has a local maximum at $(e^2,\ 8e^{-1})$.

[20 POINTS]

$8e^{-1} = ax^b \ln(e^2)$

$\frac{8}{e} = ax^b \cdot 2$

$\frac{8}{e} = a(e^2)^b \cdot 2$

$\frac{8}{e} = 2e^{2b} a$

$\frac{4}{e} = e^{2b} a$

$4e^{-1} = e^{2(-\frac{1}{2})} a$

$4e^{-1} = e^{-1} a$

$\boxed{4 = a}$

$y' = abx^{b-1} \ln x + ax^b \cdot \frac{1}{x}$

$0 = ab(e^2)^{b-1} \cdot \ln(e^2) + a(e^2)^b \cdot \frac{1}{e^2}$

$0 = abe^{2b-2} \cdot 2 + ae^{2b} \cdot e^{-2}$

$0 = 2abe^{2b-2} + ae^{2b-e}$

$0 = ae^{2b-2}(2b+1)$

$2b+1 = 0$

$\boxed{b = -\frac{1}{2}}$

(2) A circular ring of wire of radius r_0 lies in a plane perpendicular to the x-axis and is centered at the origin. The ring has a positive electric charge spread uniformly over it. The electric field in the x-direction (E at the point x on the axis) is given by $E = \frac{kx}{(x^2 + r_0^2)^{3/2}}$ for $k > 0$.

At what point on the x-axis is the field greatest? At what point on the x-axis is the field least?

[20 POINTS]

$E = kx(x^2 + r_0^2)^{-3/2}$

$E' = k(x^2+r_0^2)^{-3/2} + kx\left[-\frac{3}{\cancel{2}}(x^2+r_0^2)^{-5/2}\right](\cancel{2}x)$

$= k(x^2+r_0^2)^{-5/2}\left[(x^2+r_0^2) - 3x^2\right] = 0$

$-2x^2 + r_0^2 = 0$

$x^2 = \frac{1}{2} r_0^2$

$x = \pm \frac{\sqrt{2}}{2} r_0$

$E = \frac{k\frac{\sqrt{2}}{2} r_0}{(\frac{1}{2}r_0^2 + r_0^2)^{3/2}} = \frac{\frac{\sqrt{2}}{2} k r_0}{(\frac{3}{2}r_0^2)^{3/2}}$

$= \frac{\frac{\sqrt{2}}{2} k}{\sqrt{\frac{27}{8}} r_0^3} > 0$